Konzultace 8. a 9. přednáška

$$f[n] \xrightarrow{\mathcal{Z}} F(z)$$

$$\mathcal{Z}\{f[n]\} = F(z)$$

$$F(z) = \sum_{n=0}^{\infty} f[n] \cdot z^{-n}$$

$$f(t) \xrightarrow{\mathcal{L}} F(p)$$

$$F(p) = \int_0^{\infty} f(t) \cdot e^{-pt}\,dt$$

I. Linearita

$$\mathcal{Z}\{a \cdot f[n] \pm b \cdot g[n]\} = \mathcal{Z}\{a \cdot f[n]\} \pm \mathcal{Z}\{b \cdot g[n]\} = a \cdot \mathcal{Z}\{f[n]\} \pm b \cdot \mathcal{Z}\{g[n]\}$$

II. Změna měřítka

$$\mathcal{Z}\{a^n f[n]\} = F\left(\frac{1}{a} z\right)$$

III. Věta o posunutí

a) $\mathcal{Z}\{f[n-m]\} = z^{-m} \mathcal{Z}\{f[n]\} = z^{-m} \cdot F(z)$ $\quad \forall n-m<0 ;\ f[n-m]=0$

b) $\mathcal{Z}\{f[n+m]\} = z^{m}\left[\mathcal{Z}\{f[n]\} - \sum_{k=0}^{m-1} f[k] z^{-k}\right] =$

$$= z^{m}\left[F(z) - \sum_{k=0}^{m-1} f[k] z^{-k}\right]$$

př.

$$\mathcal{Z}\{f[n-2]\} = z^{-2} \cdot F(z)$$

$$\mathcal{Z}\{f[n+2]\} =$$

$$= z^2\left[F(z) - \sum_{k=0}^{1} f[k] \cdot z^{-k}\right] = z^2\left[F(z) - \left(f[0] \underbrace{z^{-0}}_{=1} + f[1] \cdot z^{-1}\right)\right] =$$

$$= z^2 F(z) - z^2\left(f[0] + f[1] \cdot z^{-1}\right) =$$

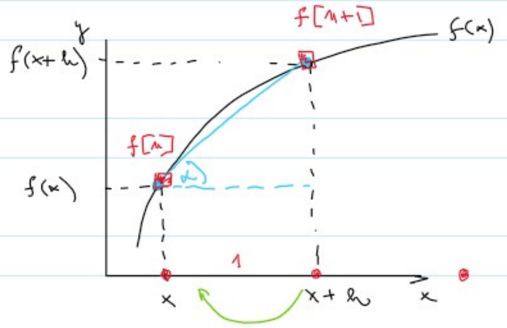
$$= z^2 F(z) - z^2 \cdot f[0] + f[1] \cdot z$$

IV. částečný součet

$$\mathcal{Z}\left\{\sum_{k=0}^{n} f[k]\right\} = \frac{z}{z-1} F(z)$$

$$\mathcal{Z}\left\{\sum_{k=0}^{n-1} f[k]\right\} = \frac{1}{z-1} F(z)$$

V. Diference



$$tg\,\alpha = \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$\Delta^1 f[n] = f[n+1] - f[n] \quad \leftarrow$ Diference

$$\Delta^0 f[n] = f[n]$$

$$\Delta^1 f[n] = f[n+1] - f[n]$$

$\Delta^2 f[n] = \Delta^1(\Delta^1 f[n]) = \Delta^1(f[n+1] - f[n]) \longrightarrow \Delta^1 f[n] = f[n+1] - f[n]$

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$$\Delta^1 f[n+1] = f[n+2] - f[n+1]$$

$$= f[n+2] - f[n+1] - (f[n+1] - f[n]) = f[n+2] - 2f[n+1] + f[n]$$

$\vdots$

$$\Delta^m f[n] = \Delta^1 [\Delta^{m-1} f[n]]$$

$$\mathcal{Z}\{\Delta^1 f[n]\} = \mathcal{Z}\{f[n+1] - f[n]\} = \Big\{ \mathcal{Z}\{f[n+m]\} = z^m \Big[F(z) - \sum_{k=0}^{m-1} f[k]\cdot z^{-k}\Big]$$

$$= \mathcal{Z}\{f[n+1]\} - \mathcal{Z}\{f[n]\} = zF(z) - z\cdot f(0) - F(z) = F(z)(z-1) - z\cdot f[0]$$

$$\Big\{ \mathcal{Z}\{f[n+1]\} = z^1\Big[F(z) - \sum_{k=0}^{0} f[k]\cdot z^{-k}\Big] = zF(z) - z\cdot f[0]\cdot z^{-0}$$

$$\mathcal{Z}\{\Delta^2 f[n]\} = \mathcal{Z}\{f[n+2] - 2\cdot f[n+1] + f[n]\} =$$

$$\Big\{ \mathcal{Z}\{f[n+2]\} = z^2\cdot[F(z) - f[0]\cdot z^{-0} - f[1]\cdot z^{-1}] = z^2 F(z) - z^2 f[0] - z\cdot f[1]$$

$$= z^2 F(z) - z^2 f[0] - z\cdot f[1] - 2[zF(z) - z\cdot f[0]] + F(z) = \ldots$$

V. Konvoluce

$$F(z) = \mathcal{Z}\{f[n]\} \; ; \; G(z) = \mathcal{Z}\{g[n]\}$$

$$\mathcal{Z}\{f[n] * g[n]\} = \mathcal{Z}\Big\{\sum_{m=0}^{\infty} f[n-m]\, g[m]\Big\} = F(z)\cdot G(z)$$

VI. Obraz derivace

$$\mathcal{Z}\{n\, f[n]\} = -z\frac{\partial F(z)}{\partial z} \qquad \Big\{ \mathcal{Z}\{f[n]\} = F(z)$$

TABULKY

$\mathbf{f(n)} = \mathcal{Z}^{-1}\{F(z)\}$	$\mathbf{F(z)} = \mathcal{Z}\{f(n)\}$	
$\delta(n)$	1	1
$\mathbf{1}(n)$	$\frac{1}{1-z^{-1}}$	$\frac{z}{z-1}$
a^n	$\frac{1}{1-az^{-1}}$	$\frac{z}{z-a}$
na^{n-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$	$\frac{z}{(z-a)^2}$
$(n+1)a^n$	$\frac{1}{(1-az^{-1})^2}$	$\frac{z^2}{(z-a)^2}$
n	$\frac{z^{-1}}{(1-z^{-1})^2}$	$\frac{z}{(z-1)^2}$
n^2	$\frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$	$\frac{z(z+1)}{(z-1)^3}$

$$\frac{1}{1-z^{-1}} = \frac{1}{1-\frac{1}{z}} = \frac{1}{\frac{z-1}{z}} = \frac{z}{z-1}$$

Inverzní Z-transformace

$$Y(z) = \frac{Q(z)}{N(z)} \quad ; \quad \text{rozklad na parciální zlomky}$$

Nabídka poptávka

$$n[k] = C\,c[k-1] + A\cdot x[k]$$
$$p[k] = -D\,c[k] + B\cdot x[k]$$

$$n[k] = p[k]$$
$$C\cdot c[k-1] + A\cdot x[k] = -D\cdot c[k] + B\cdot x[k]$$

$$D\cdot c[k] + C\,c[k-1] = B\cdot x[k] - A\cdot x[k]$$
$$c[k] + \frac{C}{D}\cdot c[k-1] = \frac{B-A}{D}\cdot x[k] \qquad \left\{ \frac{C}{D} = \gamma \;,\; \frac{B-A}{D} = \alpha \right.$$

$$y[k] + \gamma\, y[k-1] = \alpha\cdot x[k] \qquad c[k] \sim y[k]$$

$$y[-1] = 0 \;;\quad x[k] = \mathbb{1}[k]$$

a) Iterace

$k=0$: $$y[0] + \gamma\cdot \cancel{y[-1]} = \alpha\cdot\mathbb{1}[0]$$ 0
$$y[0] = \alpha\cdot\mathbb{1}[0] = \alpha$$

$k=1$: $$y[1] + \gamma\cdot y[0] = \alpha\cdot\mathbb{1}[1]$$
$$y[1] + \gamma\cdot\alpha = \alpha \Rightarrow y[1] = \alpha - \gamma\cdot\alpha = \alpha(1-\gamma)$$

$k=2$: $$y[2] + \gamma\cdot y[1] = \alpha\cdot\mathbb{1}[2]$$
$$y[2] + \gamma(\alpha(1-\gamma)) = \alpha$$
$$y[2] = \alpha - \gamma(\alpha(1-\gamma)) = \alpha - \gamma(\alpha - \alpha\cdot\gamma) =$$
$$= \alpha - \gamma\cdot\alpha + \gamma^2\cdot\alpha$$

$\vdots$

$$y[n] = \alpha\left(1 - \gamma + \gamma^2 - \gamma^3 + \gamma^4 \cdots (-\gamma)^n\right)$$
$$= \alpha\cdot\sum_{m=0}^{n}(-\gamma)^m = \alpha\cdot\frac{1-(-\gamma)^{n+1}}{1+\gamma} =$$
$$= \frac{\alpha}{1+\gamma} - \frac{\alpha(-\gamma)^{n+1}}{1+\gamma} \qquad \left\{ (-\gamma)^{n+1} = (-\gamma)\cdot(-\gamma)^n \right\}$$
$$= \boxed{\frac{\alpha}{1+\gamma} + \frac{\alpha\gamma}{1+\gamma}\cdot(-\gamma)^n}$$

b) Z-transformace

$$y[n] + \gamma\cdot y[n-1] = \alpha\cdot\mathbb{1}[n] \;;\quad y[-1]=0 \;;\quad y[0]=0$$
$$Y(z) + \gamma\cdot z^{-1}\,Y(z) = \alpha\cdot\frac{1}{1-z^{-1}}$$

$\mathbf{1}(n)$	$\frac{1}{1-z^{-1}}$	$\frac{z}{z-1}$

$$Y(z)\left(1+\gamma\cdot z^{-1}\right) = \frac{\alpha}{1-z^{-1}}$$
$$Y(z) = \frac{\alpha}{(1-z^{-1})(1+\gamma\cdot z^{-1})} = \frac{A}{1-z^{-1}} + \frac{B}{1+\gamma\cdot z^{-1}} \qquad 1+\frac{\gamma}{z} = 0$$
$$z = -\gamma$$

$$A = \frac{\alpha}{(\cancel{\quad})(1+\gamma\cdot z^{-1})}\Bigg| = \frac{\alpha}{1+\gamma} \qquad 1 - \frac{1}{z} = 0 \Rightarrow z = 1$$

$$A = \frac{\alpha}{(\;)(1+\gamma \cdot z^{-1})}\Bigg|_{z=1} = \frac{\alpha}{1+\gamma} \qquad 1-\frac{1}{z} = 0 \Rightarrow z = 1$$

$$B = \frac{\alpha}{(1-z^{-1})(\;)}\Bigg|_{z=-\gamma} = \frac{\alpha}{1-\frac{1}{(-\gamma)}} = \frac{\alpha}{\frac{\gamma+1}{\gamma}} = \frac{\alpha \cdot \gamma}{1+\gamma}$$

$$Y(z) = \frac{\alpha}{1+\gamma} \cdot \frac{1}{1-z^{-1}} + \frac{\alpha \cdot \gamma}{1+\gamma} \cdot \frac{1}{1+\gamma \cdot z^{-1}}$$

$$\underline{y[n] = \frac{\alpha}{1+\gamma} \cdot \mathbb{1}[n] + \frac{\alpha \cdot \gamma}{1+\gamma} \cdot (-\gamma)^n}$$

a^n	$\frac{1}{1-az^{-1}}$	$\frac{z}{z-a}$

Konzultace

Přenosová funkce II

$h(t)$ a) $x(t) = e^{2t} \longrightarrow \boxed{\quad} \longrightarrow y(t) = \frac{1}{6} e^{2t}$ $\quad t > 0$

b) $h'(t) + 2h(t) = e^{-4t} \cdot \underbrace{\mathbb{1}(t)}_{=1} + \beta \cdot \mathbb{1}(t)$

$\boxed{H(p)}$

nulové p.p.

a) $x(t) = u(t) = e^{2t}$; $\mathcal{L}\{u(t)\} = \frac{1}{p-2} \rightarrow U(p)$

$y(t) = \frac{1}{6} \cdot e^{2t}$; $\mathcal{L}\{y(t)\} = \frac{1}{6} \cdot \frac{1}{p-2} \rightarrow Y(p)$

$$H(p) = \frac{Y(p)}{U(p)} = \frac{\frac{1}{6} \cdot \frac{1}{p-2}}{\frac{1}{p-2}} = \underline{\underline{\frac{1}{6}}}$$

b) $p \cdot H(p) - \cancel{h(0)} + 2 \cdot H(p) = \mathcal{L}\{e^{-4t}\} + \mathcal{L}\{\beta \cdot \mathbb{1}(t)\}$

$$p \cdot H(p) + 2H(p) = \frac{1}{p+4} + \frac{\beta}{p}$$

$$H(p)(p+2) = \frac{p + \beta p + 4\beta}{p(p+4)}$$

$$H(p) = \frac{p + \beta p + 4\beta}{p(p+2)(p+4)} = \frac{(1+\beta)p + 4\beta}{p(p+2)(p+4)}$$